Formula of the century E=mc2, that we do not understand.

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Spassky Stanislav.
stanislav@spassky.net

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So, the formula of the century, $E=mc^2$. Published in Einstein's article in 1905, printed soon after Einstein's article from which it is conventional to count the beginning of the STR. The article itself (reproduced here) is not the easiest one to read, so you may limit yourself to brief parallel explanations. Or you may assess the original source. Further we will deduce the formula based on practically the same scheme, without using the STR. Also we will show that the essence and meaning of the formula is somewhat different. So:

DOES THE INERTIA OF A BODY DEPEND UPON ITS ENERGY-CONTENT?

By A. Einstein
September 27, 1905

The results of the previous investigation lead to a very interesting conclusion, which is here to be deduced.

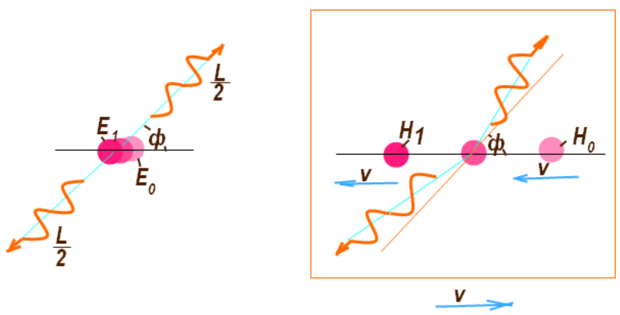
I based that investigation on the Maxwell-Hertz equations for empty space, together with the Maxwellian expression for the electromagnetic energy of space, and in addition the principle that:--

The laws by which the states of physical systems alter are independent of the alternative, to which of two systems of coordinates, in uniform motion of parallel translation relatively to each other, these alterations of state are referred (principle of relativity).

With these principles* as my basis I deduced *inter alia* the following result (§ 8):--

Initially Einstein recalls the principle of relativity.

Then there is considered one and the same simple situation in two reference systems. The situation itself reduces to a body with a mass, which, when viewed in the system of the body itself, emits to opposite sides 2 similar photons. The photons' direction line angle φ is, in principle, arbitrary, and their total energy is L. The body itself after the emission, obviously, remains where it was.



The 2nd system, in which the same situation is considered, moves (in our drawing right) at velocity v. In this system the body, as it should be, moves left at velocity v, then emits 2 photons and, according to the previous situation in the 1st system, continues to move left at the previous velocity v.

In the moving system the directions of the photons already do not look opposite. Each of the 2 direction vectors acquires an increment (and turn) left due to the system's movement right, as it is seen in the drawing.

Then follow the energy balance formulas in both systems.

In this fragment of the article pay attention to the energy conversion formulas of each photon according to the STR at transition into the moving system, and then the same for the total their energy, which increases by Lorentz factor.

With these principles* as my basis I deduced *inter alia* the following result (§ 8):--

Let a system of plane waves of light, referred to the system of co-ordinates (*x, y, z*), possess the energy *l*; let the direction of the ray (the wave-normal) make an angle ϕ with the axis of x of the system. If we introduce a new system of co-ordinates (ξ, η, ζ) moving in uniform parallel translation with respect to the system (*x, y, z*), and having its origin of co-ordinates in motion along the axis of x with the velocity *v*, then this quantity of light--measured in the system (ξ, η, ζ)--possesses the energy

$$l^* = l\frac{1 - \frac{v}{c}cos\phi}{\sqrt{1 - v^2/c^2}}$$

where c denotes the velocity of light. We shall make use of this result in what follows.

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Let there be a stationary body in the system (*x, y, z*), and let its energy--referred to the system (*x, y, z*) be E_0. Let the energy of the body relative to the system (ξ, η, ζ) moving as above with the velocity *v*, be H_0.

Let this body send out, in a direction making an angle ϕ with the axis of *x*, plane waves of light, of energy ½L measured relatively to (*x, y, z*), and simultaneously an equal quantity of light in the opposite direction. Meanwhile the body remains at rest with respect to the system (*x, y, z*). The principle of energy must apply to this process, and in fact (by the principle of relativity) with respect to both systems of co-ordinates. If we call the energy of the body after the emission of light E_1 or H_1 respectively, measured relatively to the system (*x, y, z*) or (ξ, η, ζ) respectively, then by employing the relation given above we obtain

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$$\begin{aligned} E_0 &= E_1 + \frac{1}{2}L + \frac{1}{2}L, \\ H_0 &= H_1 + \frac{1}{2}L\frac{1 - \frac{v}{c}cos\phi}{\sqrt{1 - v^2/c^2}} + \frac{1}{2}L\frac{1 + \frac{v}{c}cos\phi}{\sqrt{1 - v^2/c^2}} \\ &= H_1 + \frac{L}{\sqrt{1 - v^2/c^2}}. \end{aligned}$$

By subtraction we obtain from these equations

E0 and E1 express the energies of a massive body in the system associated with it (in the 1st system) before and after the emission. H0 and H1 are the same energies in the moving system. The total energy of the photons in the 2nd system is greater by Lorentz factor $1/\sqrt{(1-v^2/c^2)}$ according to the STR. And it is regardless of angle φ. Note that this result for the couple of opposite photons is equivalent to the energy change in the STR for a massive body.

By subtraction we obtain from these equations

$$H_0 - E_0 - (H_1 - E_1) = L\left\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right\}.$$

The two differences of the form H - E occurring in this expression have simple physical significations. H and E are energy values of the same body referred to two systems of co-ordinates which are in motion relatively to each other, the body being at rest in one of the two systems (system (x, y, z)). Thus it is clear that the difference H - E can differ from the kinetic energy K of the body, with respect to the other system (ξ, η, ζ), only by an additive constant C, which depends on the choice of the arbitrary additive constants of the energies H and E. Thus we may place

depends on the choice of the arbitrary additive constants of the energies H and E. Thus we may place

$$\begin{aligned} H_0 - E_0 &= K_0 + C, \\ H_1 - E_1 &= K_1 + C, \end{aligned}$$

since C does not change during the emission of light. So we have

$$K_0 - K_1 = L\left\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right\}.$$

The kinetic energy of the body with respect to (ξ, η, ζ) diminishes as a result of the emission of light, and the

The difference of energies for a massive body in 2 systems, namely in the moving one and in the motionless one, (H-E), coincides with the notion of the kinetic energy of a body when the body acquires velocity v.

The kinetic energy of the body with respect to (ξ, η, ζ) diminishes as a result of the emission of light, and the amount of diminution is independent of the properties of the body. Moreover, the difference $K_0 - K_1$, like the kinetic energy of the electron (§ 10), depends on the velocity.

Neglecting magnitudes of fourth and higher orders we may place

$$K_0 - K_1 = \frac{1}{2}\frac{L}{c^2}v^2.$$

From this equation it directly follows that:--

If a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2. The fact that the energy

Let us sum up the result of the termwise deduction in the two systems: the difference in the kinetic energy of the massive body before the emission and after the emission is equal to the difference of energies of the photons in the two systems (the increment of the photons energy) : $K0-K1= L*(1/\sqrt{(1-v^2/c^2)} - 1)$.

When v ->0, we have: $\Delta(m*v^2/2) \approx L*(v^2/c^2)/2$. Since the velocity v in $m*v^2/2$ does not change after the emission (according to the situation in the 1st system), we see that only the mass m can change. So we have $\Delta m=L/c^2$, and the famous formula $E=mc^2$. The conclusion of the article: the mass of a body is the measure of the internal energy.
The radiation of energy must be accompanied by the change in mass. This, in principle, can be verified experimentally.

If a body gives off the energy L in the form of radiation, its mass diminishes by L/c². The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general conclusion that

The mass of a body is a measure of its energy-content; if the energy changes by L, the mass changes in the same sense by $L/9 \times 10^{20}$, the energy being measured in ergs, and the mass in grammes.

It is not impossible that with bodies whose energy-content is variable to a high degree (e.g. with radium salts) the theory may be successfully put to the test.

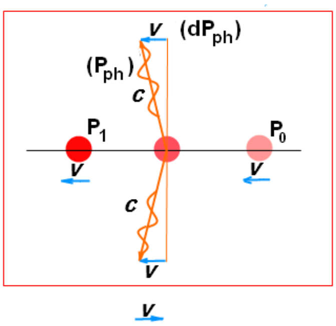
If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies.

So, that is all for article. Let us pay attention to the fact that the deduction of Einstein's formula has very little to do with the relativity theory itself. Remember, that the relationship itself at the end of the article is deduced for v tending to 0, i.e. exactly when the STR practically coincides with the classical physics. In fact, the basis for the deduction of this relationship is the situation when two kinds of particles interact – a massive particle and a pair of massless ones. In the photon with relationship between energy and momentum the situation is extremely clear, p=E/c. In the massive body everything is more complicated. With a constant mass you will not have the fulfillment of the energy and momentum conservation laws at interaction of both kinds of particles. You need to take additional energy from somewhere. And this contradiction allows redefining the notion of mass and relating it to other notions. And this is exactly what leads us to the known formula.

Let us show how the same relationship is deduced simply, without using the relativity theory.

Consider the same situation as in Einstein's article, but within the framework of the classical physics.

We can present the proof, as in Einstein's case, for any photon emission line angles. But, in order to simplify the situation, let us note that for proving and evaluation the mass change we may select any direction of motion of the 2nd system. For simplicity only, let us select direction v of the 2nd system transversal to the photons' emission line.



Also, we may, as in Einstein's case, consider the balance by energy. But for greater simplicity, let us consider the balance not by energy, but by momentum.

Due to the movement of the 2nd system right, the directions of the photons' vectors (as of the light waves) will turn left from their original directions by angle v/c (v<<c). The momentum vector of each photon Pph will also turn. It means that each photon in this system acquires an increment in momentum ΔPph to the left, equal to Pph*(v/c). Or, if by the photon energy: (Eph/c)*(v/c)=Eph*v/c². Both photons together take energy E2ph≈L from the mass and take leftward momentum ΔP2ph=E2ph*v/c²= L*v/c².

So, the mass must lose momentum ΔP=Δ(mv)=L*v/c². But let us remember that the mass velocity here does not change during the photons' emission (according to the situation in the 1st system). This is why the loss in momentum of the mass may only be explained by the loss of the mass ΔP=Δm*v=L*v/c², or Δm=L/c². And if the entire mass transforms into emission, then energy E of the entire mass (at rest) is E=mc². We obtained Einstein's formula by his own scheme, without any use of the STR.

« **Formula of the century**

The great achievement of the special theory of relativity is establishing the relationship between energy and mass... »

Myths, myths... As well as the phrase "Special theory of relativity by Einstein...", replicated both in popular and, unfortunately, in scientific literature. The STR is a collective product of many remarkable scientists.

As for Einstein, in his lectures he connected this formula with the SRT.

Let us reason:

Return to the situation in Einstein's article (a massive particle and 2 photons that it emits). Let us look at the problem at a wider and more general angle than a conclusion from some physical contradiction. Photons have quite definite relationship between energy and momentum, E=c*P. This is not that clear and explainable for a massive particle.

As you understand: 1) the solution for the massive particle by the mass, taking into account the fulfillment of energy and momentum conservation, must be the only one; 2) if we can "assemble" a massive body out of elements like photon, then this will be the noncontradictory and only solution sought for. By the way, it was not for nothing that Einstein used in the scheme exactly a pair of opposite photons. Their energy at transition into another system increases by Lorentz factor $1/\sqrt{(1-v^2/c^2)}$, just as the energy of any massive particle does. I.e. a pair of opposite photons behaves as a mass with respect of both energy and momentum. The pair is the simplest variant of a massive body, according to both classical concepts and STR. Only based on the considered conclusion was it possible to come to the STR.

Let us try to make a thought experiment. Imagine a massless box filled with photons. For this consideration it is better to imagine the entire totality of the photons as a set of opposite pairs (we have the right to do so). We obtain a wave object of type "standing wave". Splitting the object by pairs we see that the system of such pairs has E, P and m with their relationship typical for an inertial particle. And there are no contradictions in understanding of the experiment: at a certain moment one pair of all freely flies out of the envelope. Consideration in any system reveals no contradictions.

Conclusion: the particle must actually consist of wave energy, the more so that this actually confirms the unity of the nature of massless and massive particles – the unity manifested in Planck frequency for both types of particles ($E=\nu h$).

Or the mass may consist of elements "like" photon ($p=E/c$). This variant, taking into account the unity of nature, is practically unlikely.

What is the set of Lorentz systems? This is the ideal set of systems in the assumption that all objects of our world consist of waves. This is either a wave object of type "travelling wave" (TrW-object), or of type "standing wave" (StW-object), i.e. a "travelling wave", but in any event closed upon itself. In those Lorentz systems that are actually at rest with respect to the ether, the space-time structure has a real nature. In other Lorentz systems, the space-time structure has somewhat artificial nature. But all Lorentz systems are symmetrical and indiscernible for wave objects since in all of them the wave objects "look" EXACTLY AS in the systems actually resting with respect to the wave medium. This is because the classical wave operator in all Lorentz systems has the same and simplest form. Full identity of the systems refers both to the wave propagation velocity in all directions and all physics of these wave objects. In the context of the conservation laws and common sense, it would be logical to select for the wave objects: wave energy scalar E and wave energy flow vector FlowE=E*c. But historically, instead of FlowE vector $p=FlowE/c^2$ was used.

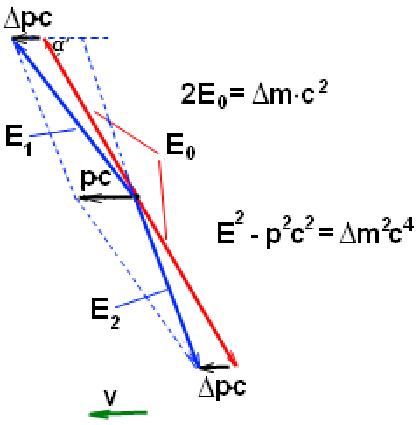
Retrospective journey into the history of notion “mass”.

Historically in classical physics mass defined as follows. First of all, as a measure of quantity of substance, secondly as a coefficient of inertia showing how much effort should be applied (i.e. F*t) in order to impart velocity v to a body: (F*t=m*v). Historically, Einstein’s formula was already known before, but instead of m it included either the so-called “electromagnetic mass”, or the quantity (mass) of carried ether, etc. In Einstein’s formula this mass is given in the form we are accustomed to. The formula was obtained purely formally. It seems to be its plus. Einstein calls mass an **equivalent** of energy, a **measure** of content of some internal energy. But when Einstein got down to the General theory of relativity, i.e. gravity, he already had to postulate the equivalence of “inertial mass” and “gravitational mass”. Now the physicists are persistently looking for some particle that is responsible for the inertial properties of all other particles. It is evident that something is wrong with the “customary notion” of mass. We should probably simply understand that localized wave energy by itself manifests inertial properties (just as gravitational ones).

Gravity turns the trajectory (and momentum) of the photons to the center of gravity, and without changing their energy, as well as all linear components of StW-object. So, StW-object must be pulled by the gravitational field, and also without changing their energy.

The proposed model of "mass", which reproduces all the properties of mass, can be used in the justification of quantum mechanics with its de Broglie wavelength. Because of the change in frequency of two opposite longitudinal to the motion components.

All that we have discussed once again confirms the unity of nature of both particle types.



At the end we will show how in classical physics, at v<<c, one can deduce the known formula of the STR $E^2 - p^2c^2 = \Delta m^2c^4$ for a pair of photons (and set of them!). Initially we have a pair of opposite similar photons with total energy 2E0=Δmc² (red color). There is no resultant momentum at rest. When considering this pair in motion, the directions of initially opposite photons will turn to the left. The pair acquires momentum p. Δp= p0(v/c)= (E0/c)*(v/c) = E0*v/c². p=Δmv. The total energy increases in motion and becomes equal to E1+E2. The photons’ direction line angle α is given with respect to vector v in the 1st system. Then we use the so-called “cosine formula” and approximation for A>>b: √(A+b) ≈√A +(b/2)/√A .

E1=√(E0²+(Δpc)² +2Cos(α)E0*Δpc) ≈ √(E0²+(Δpc)²) +Cos(α)E0*Δpc/√(E0²+(Δpc)²)
E2=√(E0²+(Δpc)² - 2Cos(α)E0*Δpc) ≈ √(E0² +(Δpc)²) - Cos(α)E0*Δpc/√(E0²+(Δpc)²)

When you add, the right terms are annihilated: E=E1+E2≈2*√ (E0² +(Δpc)²)
E²=(2E0)²+(2Δpc)²   **E²=(Δmc²)²+(pc)²**

We obtained the formula $E^2 - p^2c^2 = \Delta m^2c^4$**,** which is especially evident for α=π/2 (in accordance with Pythagorean formula). In the case of set of pairs, all vectors pc in them are parallel and in all pairs the values pc, E0, E are in same proportion regardless of α.

We should pay tribute to the scheme suggested by Einstein. This scheme with emission of exactly a pair of opposite photons, and consideration of this phenomenon in two systems is original and “with a twist”.

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